

# Massive Satellites of Close-In Gas Giant Exoplanets

Timothy A. Cassidy<sup>1</sup>, Rolando Mendez<sup>2</sup>, Phil Arras<sup>2</sup>, Robert E. Johnson<sup>1,2</sup>, Michael F. Skrutskie<sup>2</sup>

tac2z@virginia.edu, rem5d@cms.mail.virginia.edu, arras@virginia.edu,  
rej@virginia.edu, mfs4n@virginia.edu

## ABSTRACT

We study the orbits, tidal heating and mass loss from satellites around close-in gas giant exoplanets. The focus is on large satellites which are potentially observable by their transit signature. We argue that even Earth-size satellites around hot Jupiters can be immune to destruction by orbital decay; detection of such a massive satellite would strongly constrain theories of tidal dissipation in gas giants, in a manner complementary to orbital circularization. The star's gravity induces significant periodic eccentricity in the satellite's orbit. The resulting tidal heating rates, per unit mass, are far in excess of Io's and dominate radioactive heating out to planet orbital periods of months for reasonable satellite tidal  $Q$ . Inside planet orbital periods of about a week, tidal heating can completely melt the satellite. We compute an upper limit to the satellite mass loss rate and show that, although rocky satellites around hot Jupiters with orbital periods less than a few days can be significantly evaporated in their lifetimes, detectable satellites can survive at longer orbital periods.

*Subject headings:*

## 1. Introduction

The high photometric precision of the Hubble Space Telescope allows interesting constraints to be placed on the existence of massive satellites orbiting transiting exoplanets. Two systems have been searched so far, HD 209458b (Brown et al. 2001) and HD 189733b

---

<sup>1</sup>Engineering Physics Program, University of Virginia  
Charlottesville, VA 22904-4325

<sup>2</sup>Department of Astronomy, University of Virginia  
P.O. Box 400325, Charlottesville, VA 22904-4325

(Pont et al. 2007). From the transit lightcurve and timing, upper limits of roughly an Earth radius and mass have been placed on hypothetical satellites around these two planets. A number of studies have discussed detection techniques (Sartoretti & Schneider 1999; Han & Han 2002; Barnes & Fortney 2004; Doyle & Deeg 2004; Szabó et al. 2006; Simon et al. 2007). This paper is concerned with the orbits and physical structure these hypothetical moons.

With the launch of the Kepler satellite (Borucki et al. 2004),  $\sim 10^5$  stars will be monitored with photometric precision sufficient to detect Earth size objects. The estimated hundreds of hot Jupiters (gas giant planets with orbital periods less than 1 week) to be found by Kepler will provide a large sample to investigate the existence of Earth-size satellites (Szabó et al. 2006).

Johnson & Huggins (2006) note another possible observational manifestation of satellites. Interaction between a satellite and the planet’s magnetosphere may give rise to an extended gaseous torus orbiting the planet, as in the Jupiter-Io system. Material stripped from the satellite’s surface and atmosphere would contribute to the transmission spectrum of the planet. It has been suggested (Vidal-Madjar et al. 2003) that absorption in the upper atmosphere of HD 209458b (Charbonneau et al. 2002; Vidal-Madjar et al. 2003, 2004; Redfield et al. 2008; Ballester et al. 2007) is due to a high rate of atmospheric escape which increases the density at large radii. The plasma torus model is an alternative explanation for such absorption high in the atmosphere.

There has been interest recently in the properties of satellites around exoplanets, mainly focused on the habitability of moons (e.g. Reynolds et al. 1987; Williams et al. 1997; Scharf 2006) around planets at large orbital separation from the star. Our focus in this paper is on planets to be found with Kepler, hence we restrict our attention on close-in planets for which the transit probability is substantially higher. As we will show, moons in such systems will be subjected to intense tidal heating, with consequences for the moon’s structure, evaporation rate, and interaction with its surroundings.

Two recent studies of the orbital stability of satellites are important in this work. Domingos et al. (2006) investigated orbital stability for point mass star, planet and satellite, for a range of eccentricity of both the planet and satellite. They point out that, for small planetary eccentricity, satellite orbits are stable for  $a_s \lesssim 0.5a_H$ , where  $a_s$  is the semi-major axis of the satellite around the planet and  $a_H$  is the planet’s Hill radius. As we will show, the satellite orbits have large periodic eccentricity induced by the star as the stability limit  $a_s \simeq a_H/2$  is approached. Hence, while Io’s eccentricity and tidal heating are due to interaction with Jupiter and the other Galilean satellites, moons of close in exoplanets have a large periodic eccentricity due to stellar forcing. We show that this periodic eccentricity leads to large tidal heating for systems close to the parent star.

As pointed out by Barnes & O’Brien (2002), such close-in moons, orbiting slowly rotating planets synchronized to the star, are subject to orbital decay. The tide raised on the planet by the moon tries to spin up the planet while shrinking the moon’s orbit, eventually leading to the moon impacting the planet. The orbital decay rate is sensitive to the tidal  $Q_p$  of the gas giant planet. Barnes & O’Brien (2002) used  $Q_p = 10^5$ , the estimate for the Jupiter-Io tidal interaction, leading to the constraint that hot Jupiters cannot have moons larger than  $\sim 10^{-4}M_\oplus$ , where  $M_\oplus$  is the Earth’s mass. Such tiny moons would be undetectable by the transit method. However, as we discuss, the appropriate value of  $Q_p$  may be larger by a factor  $\sim 10^8$ , allowing stable orbits for potentially detectable moons with radii  $\gtrsim R_\oplus$ .

Little attention has been given to the formation scenario of the massive satellites discussed in this paper. Canup & Ward (2006) note that the outer planets of the Solar System contain only a fraction  $10^{-4}$  of the planet’s mass in the satellite systems. If applied to extrasolar planets, this phenomenological scaling implies satellites of mass  $\lesssim M_\oplus/30$ , which would be undetectable by the transit method. An alternative possibility is for the gas giant to capture a terrestrial planet into a bound orbit.

Satellite orbits are discussed in section 2. Orbital decay is discussed in section 3. Tidal heating and consequences for internal structure are discussed in section 4. Coupled tidal heating and satellite orbital evolution are discussed in section 5. Evaporative mass loss is discussed in section 6 and conclusions are stated in section 7.

## 2. Satellite orbits

In this section we review the moon’s orbit, treating moon, planet and star as point masses. This problem is referred to as the “main problem in lunar theory” (Brouwer & Clemence 1961), and considerable analytic progress can be made.

We consider star, planet and satellite of masses  $M_* \gg M_p \gg M_s$ , respectively. We will ignore the influence of the satellite on the star-planet orbit, treating it as Keplerian with semi-major axis  $a_p$  and eccentricity  $e_p$ . The mean motion of the planet’s orbit is  $n_p \simeq (GM_*/a_p^3)^{1/2} = 2\pi/P_p$ . The satellite’s orbit around the planet can be treated as nearly Keplerian, with semi-major axis  $a_s$ , eccentricity  $e_s$  and mean motion  $n_s \simeq (GM_p/a_s^3)^{1/2} = 2\pi/P_s$ .

We work in a reference frame with origin at the center of the planet and non-rotating axes fixed with respect to distant observers. Cylindrical coordinates  $(r_s, \phi_s)$  are used for the satellite and  $(r_*, \phi_*)$  for the star. The orbits are assumed coplanar for simplicity. The

equations of motion for the satellite are then

$$\ddot{r}_s = \frac{\ell_s^2}{r_s^3} - \frac{GM_p}{r_s^2} + \frac{\partial \mathcal{R}}{\partial r_s} \quad (1)$$

$$\dot{\phi}_s = \frac{\ell_s}{r_s^2} \quad (2)$$

$$\dot{\ell}_s = \frac{\partial \mathcal{R}}{\partial \phi_s} \quad (3)$$

where  $\ell_s$  is the satellite's orbital angular momentum per unit mass and  $\mathcal{R} = \mathcal{R}(r_s, \phi_s, r_*, \phi_*)$  is the negative of the tidal gravitational potential from the star. We can simplify the form of  $\mathcal{R}$  using the fact that  $r_s \ll r_*$  to find the leading order result in  $r_s/r_*$  (Brouwer & Clemence 1961),

$$\mathcal{R} = \frac{GM_* r_s^2}{2r_*^3} (3 \cos^2[\phi_s - \phi_*] - 1). \quad (4)$$

Given an orbit for the planet-star system,  $r_*(t)$  and  $\phi_*(t)$ , eq.1, 2 and 3 can be integrated in time to find the orbit of the satellite.

The parameter space for stable satellite orbits is strongly constrained. For close-in planets, the planetary radius  $R_p$  can be a large fraction of the Hill radius,  $a_H = a_p(M_p/3M_*)^{1/3}$ , implying all bound satellite orbits are significantly perturbed by the star's gravity. Domingos et al. (2006) found stable orbits for  $a_s \lesssim 0.49a_H(1.0 - 1.0e_p - 0.27e_s)$ . In terms of the orbital periods, the stability limit is  $P_s \lesssim P_p/5$  for  $e_p, e_s \ll 1$ . The dimensionless strength of the disturbing potential is then  $(n_p/n_s)^2 = (P_s/P_p)^2 \leq 1/25$ . Ignoring the cohesive strength, the moon must also orbit outside the planet's Roche radius at  $a_{\text{Roche}} = 3^{1/3}R_p(M_p R_s^3/M_s R_p^3)^{1/3}$ , where  $R_p$  is the radius of the planet. For gas giants with mean density  $\rho_p = 3M_p/(4\pi R_p^3) \sim 1 \text{ g cm}^{-3}$  and terrestrial satellites with mean density  $\rho_s = 3M_s/(4\pi R_s^3) \sim 5 \text{ g cm}^{-3}$ , the Roche radius is at or below the planets surface. Hence we can consider the satellite disrupted if it hits the planet's surface. In summary, satellite orbits are bounded at short orbital periods by the planet's surface and at long orbital periods by the stability constraint, giving the range of possible orbital periods  $2\pi(R_p^3/GM_p)^{1/2} \lesssim P_s \lesssim P_p/5$ , for  $e_s, e_p \ll 1$ .

The derivation in Brouwer & Clemence (1961) uses the disturbing function formalism and computes the variation of the orbital elements  $a_s, e_s$ , etc rather than  $r_s$  and  $\phi_s$  directly. Here we give a simpler derivation of the variation involving  $r_s$  and  $\phi_s$ . We consider a circular planetary orbit with  $r_* = a_p$  and  $\phi_* = n_p t$ , and a satellite orbit described by a circular orbit plus small perturbations induced by the star. In detail, we write

$$r_s(t) = a_s + \delta r_s(t) \quad (5)$$

$$\phi_s(t) = \phi_{s0} + n_s t + \delta\phi_s(t) \quad (6)$$

$$\ell_s(t) = a_s^2 n_s + \delta\ell_s(t). \quad (7)$$

Here  $\phi_{s0}$  is a constant phase. Plugging eq.5, 6 and 7 into eq.1, 2 and 3, the equations are satisfied at leading order. Eq.3 can be immediately integrated to yield

$$\frac{\delta\ell_s(t)}{\ell_s} = \frac{3}{4} \left( \frac{n_p}{n_s} \right)^2 \cos [2(n_s - n_p)t + 2\phi_{s0}], \quad (8)$$

where we have approximated  $n_p \ll n_s$  to simplify the expression. Plugging eq.8 into eq.1, we find the following forced harmonic oscillator equation for  $\delta r$ ,

$$\delta\ddot{r}_s + n_s^2 \delta r_s = n_p^2 a_s \left( \frac{1}{2} + 3 \cos [2(n_s - n_p)t + 2\phi_{s0}] \right). \quad (9)$$

The first term on the right hand side of eq.9 gives a shift in the orbital radius  $\delta r_s = (1/2)a_s(n_p/n_s)^2$ , as if the planet’s mass was slightly smaller. This constant term causes no tidal heating, so we ignore it from here on. The second term on the right hand side of eq.9 causes an oscillatory change in  $\delta r_s$ . We can solve for this term by writing  $\delta r_s(t) = \delta r_{s0} \cos ([2(n_s - n_p)t + 2\phi_{s0}])$ , where  $\delta r_{s0}$  is a constant. The cosine factor then cancels out of the equation and the amplitude  $\delta r_{s0}$  can be found. The result is

$$\frac{\delta r_s(t)}{a_s} = - \left( \frac{n_p}{n_s} \right)^2 \cos [2(n_s - n_p)t + 2\phi_{s0}]. \quad (10)$$

Plugging eq.8 and 10 into eq.2, we find the angle

$$\delta\phi_s(t) = \frac{11}{8} \left( \frac{n_p}{n_s} \right)^2 \sin [2(n_s - n_p)t + 2\phi_{s0}]. \quad (11)$$

Eq.10 and 11 confirm that the forced orbital variations vary as  $(P_s/P_p)^2$  for small eccentricity orbits. These formulas are valid when  $P_s \ll P_p$ , and underestimate the perturbations as the orbit approaches instability. We will use eq.10 and 11 to compute the tidal heating rate in section 4.

Figure 1 compares a direct numerical integration of eq. 1, 2 and 3 against the analytic formulas in eq.10 and 11, showing that they agree to the stated accuracy in the small parameter  $P_s/P_p$ .

The discussion so far has examined small perturbations about circular planet and satellite orbits. The same method can be used to find small perturbations, induced by the star, about orbits with finite “background” eccentricities  $e_{s0}$  or  $e_{p0}$ . A detailed derivation and results can be found in Brouwer & Clemence (1961). Again orbital variations of size  $(P_s/P_p)^2$

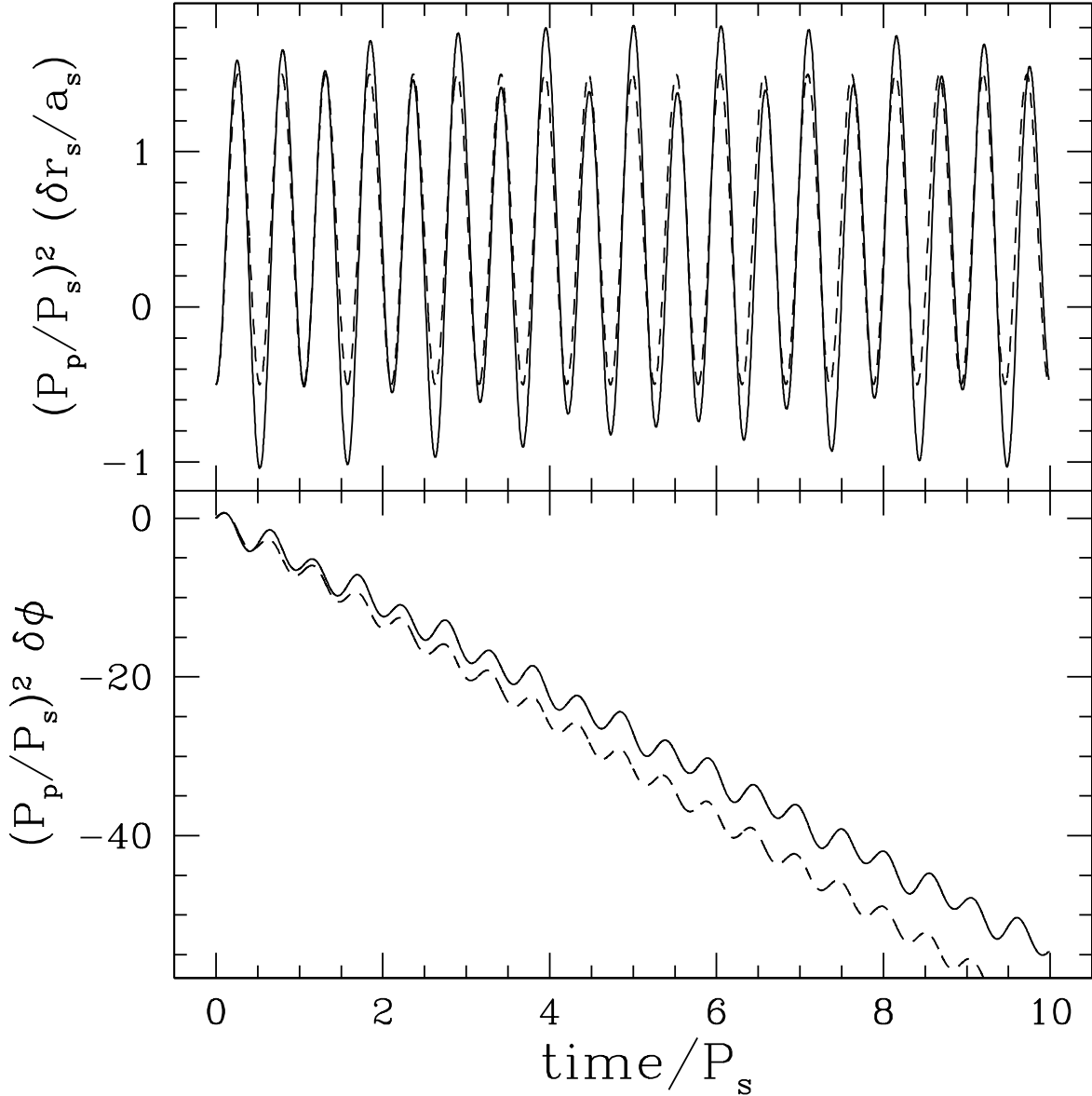


Fig. 1.— Perturbations to the radius (upper panel) and longitude (lower panel) due to the stellar gravitational field. Here  $\delta r_s = r_s(t) - a_s$  and  $\delta\phi_s = \phi_s(t) - n_s t$ . The solid lines are the result of numerical integration of eq. 1, 2 and 3 for  $P_p/P_s = 20$ , scaled by the factor  $(P_p/P_s)^2$  to make the numbers of order unity. The dashed lines are the analytic results including the periodic components from eq. 10 and 11, as well as the time-independent shift to the radius  $\delta r_s/a_s = (n_p/n_s)^2/2$ , and the resultant change in orbital period. This change in orbital period is the cause of the linear behavior of  $\delta\phi$  in the lower plot, while the oscillatory term from eq. 11 causes the short period variations.

are found due to stellar perturbations. Tidal heating of the satellite can arise due to either background  $e_{s0}$  and  $e_{p0}$ , or perturbations induced by the star, which exists even for  $e_{s0} = e_{p0} = 0$ . We expect the relative size of these two effects scales as  $e_{s0}$  or  $e_{p0}$  versus  $(P_s/P_p)^2$ . The forced variation should provide a *lower limit* to the tidal heating, and may significantly underestimate the heating for finite eccentricity orbits at large  $a_p$ .

### 3. Orbital decay due to tides raised in the gas giant planet

Tidal friction is recognized to be an important factor in the survival of hypothetical early satellite systems for inner Solar System planets Mercury and Venus (e.g., Burns 1973; Ward & Reid 1973). These studies pointed out that tides raised in the planet by the satellite and the Sun can cause significant changes in the satellite’s orbit. The problem is even more severe as the planet is moved closer to the star, as the stellar tides are stronger and the Hill sphere is closer to the planet. However, a key difference occurs if the planet in question is a gas giant rather than a rocky body. Gas giants are less dissipative than rocky bodies, and hence satellites are relatively more immune to destruction by orbital decay into the planet.

Barnes & O’Brien (2002) studied orbital decay of satellites around hot Jupiters. They point out that if the gas giant planet is synchronized to the parent star, then the satellite orbits faster than the planet spins. Tides raised in the planet by the satellite then attempt to spin up the planet while shrinking the orbit of the satellite to conserve angular momentum. The conclusion of their study was that only satellites with masses  $M_s \ll M_\oplus$  could survive being dragged down to the planet. Such tiny satellites would be undetectable by the transit method. We point out that recent work on the theory of tidal  $Q_p$  in gas giants implies that orbital decay is far less effective than the estimate by Barnes & O’Brien (2002), and orbital decay is ineffective even for Earth mass satellites.

Using the equations from Goldreich & Soter (1966), orbital decay over a timescale  $T$  will lead to destruction of satellites with mass

$$M_s \geq M_{s,\text{crit}} = 5.4 \times 10^{-3} M_p \left( \frac{Q_p P_s^{13/3}}{P_{\text{dyn}}^{10/3} T} \right), \quad (12)$$

where  $Q_p$  is the quality factor for the gas giant planet and  $P_{\text{dyn}} = 2\pi(R_p^3/GM_p)^{1/2}$  is the dynamical time of the planet (2.8 hr for Jupiter). Barnes & O’Brien (2002) evaluated this expression using  $Q_p \simeq 10^5$ , the value inferred for orbital expansion in the Jupiter-Io system (Goldreich & Soter 1966), as well as circularization of the extrasolar planets (e.g., Wu 2003),

finding

$$\begin{aligned}
 M_{s,\text{crit}} &= 5 \times 10^{-5} M_{\oplus} \left( \frac{M_p}{M_{\text{Jup}}} \right) \left( \frac{Q_p}{10^5} \right) \\
 &\times \left( \frac{P_s}{P_p/5} \frac{P_p}{4 \text{ days}} \right)^{13/3} \left( \frac{2.8 \text{ hr}}{P_{\text{dyn}}} \right)^{10/3} \left( \frac{5 \text{ Gyr}}{T} \right), \tag{13}
 \end{aligned}$$

where  $M_{\text{Jup}}$  is the mass of Jupiter, and we have scaled  $P_s$  in terms of the maximum orbital period  $P_p/5$ . If true, this result would imply that Earth mass satellites would only be stable outside orbital periods of  $\sim 40$  days, where the transit probability is much smaller.

The origin of the tidal  $Q_p$  in gas giant planets has been a mystery for decades. Goldreich & Nicholson (1977) pointed out that turbulent eddies in the planet have long turnover times,  $t_{\text{eddy}}$ , compared to the forcing periods of interest, severely decreasing the turbulent viscosity. They estimated  $Q_p \sim 10^{13}$  for “equilibrium-tide” flow in Jupiter. This disagreement between theory and observation stood for several decades. Recent work by Ogilvie & Lin (2004) and Wu (2005) has found that when the tidal forcing frequencies are resonant with inertial waves (waves with frequencies less than twice the spin frequency, restored by the Coriolis force) that  $Q_p \ll 10^{13}$  can be obtained, due to the higher dissipation rate for short lengthscale inertial waves. Their work has clearly shown that resonant excitation of inertial waves may be the key to understanding the small observed  $Q_p$  values.

Such small  $Q_p$  values do not apply to tides raised in a slowly rotating planet by the satellite. If synchronized, the spin period of the gas giant equals its orbital period,  $P_p$ . But the orbital period of the satellite  $P_s \leq P_p/5$  for orbital stability. For a quadrupole tide raised in the planet, the forcing period would be shorter than  $P_p/10$ , while inertial waves only exist with periods longer than  $P_p/2$ . Hence inertial waves cannot be excited in synchronous planets by the satellite and  $Q_p \sim 10^5$  does not apply. The forcing frequency is also far too low to be resonant with the f-mode (periods  $\sim$  a few hours), hence no normal modes of oscillation are available for resonance with the tide.

The estimate of Goldreich & Nicholson (1977) found that dissipation due to turbulent viscosity was concentrated just below the tropopause, near  $\sim 1$  bar for Jupiter. For the hot Jupiters, strong insolation pushes the tropopause down to  $\sim$ kbar pressures (e.g., Arras & Bildsten 2006), where  $t_{\text{eddy}}$  is much longer. We will use Goldreich and Nicholson’s estimate of  $Q = 10^{13}$ , keeping in mind that it may underestimate  $Q$  (overestimate dissipation). Plugging this value into eq.13 we find

$$\begin{aligned}
 M_{s,\text{crit}} &= 5000 M_{\oplus} \left( \frac{M_p}{M_{\text{Jup}}} \right) \left( \frac{Q_p}{10^{13}} \right) \\
 &\times \left( \frac{P_s}{P_p/5} \frac{P_p}{4 \text{ days}} \right)^{13/3} \left( \frac{2.8 \text{ hr}}{P_{\text{dyn}}} \right)^{10/3} \left( \frac{5 \text{ Gyr}}{T} \right). \tag{14}
 \end{aligned}$$

Hence orbital decay is ineffective for Earth mass satellites at all orbital periods of interest, and orbital decay of satellites around synchronous gas giant planets can be ignored, as these planets are so weakly dissipative.

#### 4. Heating of the satellites of close-in exoplanets

First we review tidal heating in Io. Based on the significant forced eccentricity of Io, Peale et al. (1979) predicted widespread surface volcanism, which was soon confirmed by images from Voyager 1 (Masursky et al. 1979). The total dissipation rate for tidal forcing of a homogeneous, incompressible, elastic sphere is (Peale & Cassen 1978; Peale et al. 1979; Wisdom 2004)

$$\dot{E} = \frac{42}{19} \frac{\pi \rho_s^2 n_s^5 R_s^7 e_{s0}^2}{\mu Q_s} \quad (15)$$

where  $\mu \simeq 6.5 \times 10^{11}$  dyne  $\text{cm}^{-2}$  is the rigidity, and  $Q_s$  is the tidal dissipation coefficient of Io. For Io,  $P_s = 1.7$  days,  $\rho_s = 3.5$  g  $\text{cm}^{-3}$ , and  $R_s = 1800$  km. The forced eccentricity of Io was determined to be  $e_{s0} = 0.0043$  (Peale et al. 1979), resulting in a dissipation rate for Io of

$$\dot{E}_{\text{Io}} = \frac{1.6 \times 10^{21}}{Q_s} \text{ erg s}^{-1}. \quad (16)$$

Peale et al. (1979) used a fiducial value  $Q_s = 100$ , based on laboratory studies on rock samples. For  $Q_s = 100$ , eq.16 gives a heating rate several times the radioactive heating rate estimate for the Moon (Peale et al. 1979). Tidal heating is widely recognized as an important effect in the thermal structure of Io.

To assess the role of tidal heating for the satellites around hot Jupiters, we first ask what would happen to an Io-like satellite orbiting around a Jupiter-like planet, which itself is in 4 day orbit around a solar-type star. As discussed in section 3, orbits can be thought of as a Keplerian ellipse with constant eccentricity  $e_{s0}$ , with perturbations due to the star’s gravity superposed. The calculation by Peale et al. (1979) includes only the contribution from  $e_{s0}$ . We will compute the opposite limit of stellar perturbations with  $e_{s0} = 0$  in this section. This can be regarded as a minimum heating rate which becomes larger for finite  $e_{s0}$ .

From eq.10 and 11, the perturbations correspond to eccentricity variations  $\sim (P_s/P_p)^2 \lesssim 1/25 = 0.04$ . For orbits near the stability limit, the forced eccentricity is larger than the forced eccentricity of Io by a factor of  $0.04/0.0043 = 9$ . The orbital period of the satellite must be smaller than  $4 \text{ days}/5 = 19$  hr, shorter than Io’s orbital period by a factor of  $\simeq 2.1$ .

Plugging into eq.15, the higher eccentricity and shorter orbital period combine to increase the heating rate by a factor  $\sim 10^4$ . Clearly tidal heating will be important for satellites around hot Jupiters even for satellite size much smaller than Io.

Tidal heating rates for satellites are sensitively dependent on the tidal  $Q_s$ . The value  $Q_s \sim 10$  for Earth-Moon orbital evolution is likely due to dissipation in shallow seas (e.g. Jeffreys 1970). Heat from such dissipation depends on the uncertain ocean depth and topography, and is easily radiated away. We focus instead on heating of the interior, especially for early stages to determine if the satellite is melted. Decay of seismic body waves with periods of  $10^3 - 10^4$  s in the Earth finds  $Q_s \sim 300 - 500$  (e.g. Roult et al. 2006), but extrapolation of this data to  $\sim 10^5 - 10^6$  s is uncertain (e.g. Burns 1977; Efroimsky & Lazarian 2000). We will use a fiducial value of  $Q_s = 100$ , assuming moderately increased dissipation when the period is extrapolated over 10 – 100 longer timescales than is measured. For a completely molten satellite,  $Q_s$  may be much larger. For instance, the Preliminary Reference Earth Model, constructed from seismological data, finds  $Q_s \sim 10^5$  for the Earth’s liquid outer core (Dziewonski & Anderson 1981).

To discuss the tidal heating rate for satellites with forced periodic eccentricity, we will derive an analytic estimate by plugging eq.10 and 11 into the more general formula found in Wisdom (2004). The tidal heating rate for a general orbit is expressed by Wisdom (2004) as

$$\dot{E} = \frac{\rho_s h_s R_s^2}{g_s} \int d\Omega U \frac{dU'}{dt} \quad (17)$$

where  $h_s = (5/2)/(1 + 19\mu/2\rho_s g_s R_s)$ ,  $g_s = GM_s/R_s^2$ ,  $U$  is the tidal potential in the satellite due to the planet,  $U'$  is the tidal potential including the dissipative lag, and the integral extends over the surface of the satellite. We perform several operations to simplify eq.17. First, we expand  $U$  in spherical harmonics and perform the angular integration. Next, we expand to leading order in the lag time  $(n_s Q_s)^{-1}$ . We assume the satellite is synchronized when evaluating the libration term. Lastly, we plug in the small deviations from a circular orbit found in eq.10 and 11. We find the result

$$\dot{E} = \frac{2817}{160} \frac{GM_p^2 h_s R_s^5 n_s}{Q_s a_s^6} \left(\frac{n_p}{n_s}\right)^4. \quad (18)$$

We expect a smooth transition between the eccentricity-dominated case given in eq.15 and the periodic variation dominated case in eq.18. Equating these two formulas we find a critical eccentricity  $e_{s,crit} \simeq 1.6(P_s/P_p)^2 \lesssim 0.06$ .

To numerically evaluate eq.18, we again use  $\mu \simeq 6.5 \times 10^{11}$  dyne  $\text{cm}^{-2}$  for the rigidity, and scale all quantities to Earth values. Since the heating rate increases as  $P_s$  decreases, we

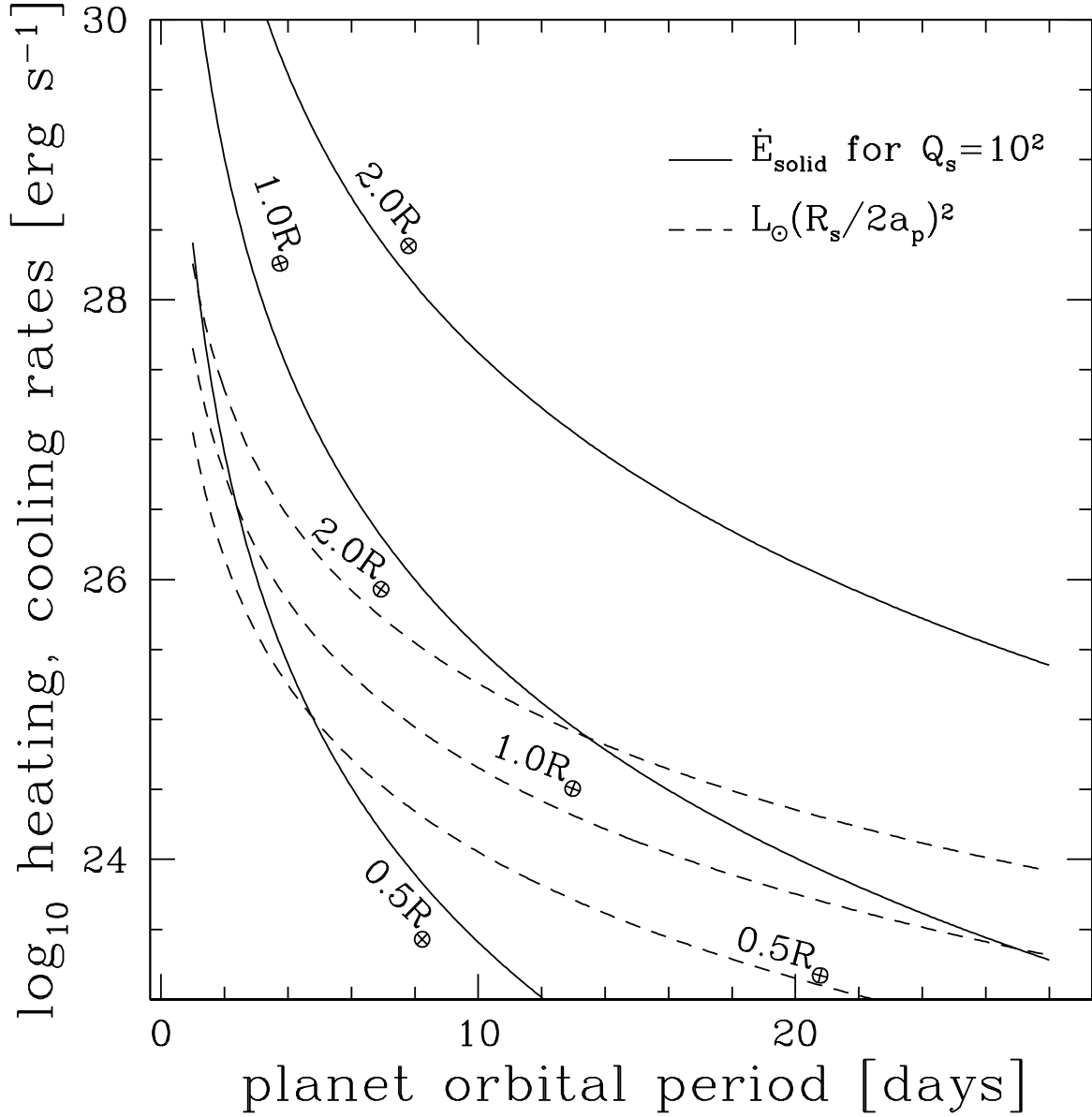


Fig. 2.— Satellite tidal heating rates (solid lines) as a function of orbital period. Eq.19 was used for the heating rate with  $\rho_s = 5 \text{ g cm}^{-3}$ ,  $Q_s = 10^2$  and  $P_s = P_p/5$ . Insolation rates (dashed lines) for a Sun-like star are shown for comparison.

scale expressions to the *minimum* heating rate using  $P_s = P_p/5$ , finding

$$\begin{aligned} \dot{E}_{\text{solid}} &= \frac{3.2 \times 10^{29} \text{ erg s}^{-1}}{Q_s} \left( \frac{\rho_s}{5 \text{ g cm}^{-3}} \right)^2 \\ &\times \left( \frac{R_s}{R_{\oplus}} \right)^7 \left( \frac{4 \text{ days}}{P_p} \right)^5 \left( \frac{P_p/5}{P_s} \right) \end{aligned} \quad (19)$$

for a solid satellite and

$$\begin{aligned} \dot{E}_{\text{liquid}} &= \frac{2.7 \times 10^{29} \text{ erg s}^{-1}}{Q_s} \\ &\times \left( \frac{R_s}{R_{\oplus}} \right)^5 \left( \frac{4 \text{ days}}{P_p} \right)^5 \left( \frac{P_p/5}{P_s} \right) \end{aligned} \quad (20)$$

for a satellite which has been completely melted ( $\mu = 0$ ). Eq.19 is compared to insolation in fig.2. While the prefactors in eq.19 and 20 are comparable, their scaling with  $R_s$  differs, and the tidal  $Q_s$  may be quite different for molten and solid satellites. If the tidal  $Q_s$  in the liquid phase is far larger than for the solid phase, then the rate of tidal heating could drop drastically upon melting.

Would Earth-size satellites around hot Jupiters be melted by tidal heating? Eq.19 implies a heating rate  $\dot{E}_{\text{solid}} \sim 10^{28} \text{ erg s}^{-1}$  for a 4 day orbital period. For a latent heat  $L = 10^{10} \text{ erg g}^{-1}$  appropriate for iron at high pressure (Stevenson et al. 1983), melting would occur in a few hundred years. The tidal heating rate, per gram, scales as  $R_s^4$ , so that heating is more intense for larger satellites. Assuming the radioactive heating rate of the Earth,  $\sim 10^{20} \text{ erg s}^{-1}$ , tidal heating dominates radioactive heating for  $P_p \lesssim 150$  days. Radioactive heating will eventually dominate for satellites far smaller than an Earth radius. Thermal models of the young Earth (Stevenson et al. 1983) typically show a completely liquid core for  $\dot{E} \gtrsim 10^{20} \text{ erg s}^{-1}$ ; the tidal heating rate is greater by many orders of magnitude. We conclude that the Earth-size satellites of hot Jupiters, observable by their transits, would be largely molten, except perhaps for a thin layer near the surface, cooled by radiation (e.g., Peale et al. 1979). If  $Q_s$  drops precipitously for complete melting, the temperature could be regulated near the melting point.

The effective temperature for the tidal heating energy flux may be found by equating the heating rate in eq.19 to the blackbody cooling rate  $4\pi R_s^2 \sigma T_{\text{eff}}^4$ , giving

$$\begin{aligned} T_{\text{eff}} &= 1800 \text{ K} \left( \frac{\rho_s}{5 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{R_s}{R_{\oplus}} \right)^{5/4} \\ &\times \left( \frac{4 \text{ days}}{P_p} \right)^{5/4} \left( \frac{P_p/5}{P_s} \right)^{1/4} \left( \frac{10^2}{Q_s} \right)^{1/4}. \end{aligned} \quad (21)$$

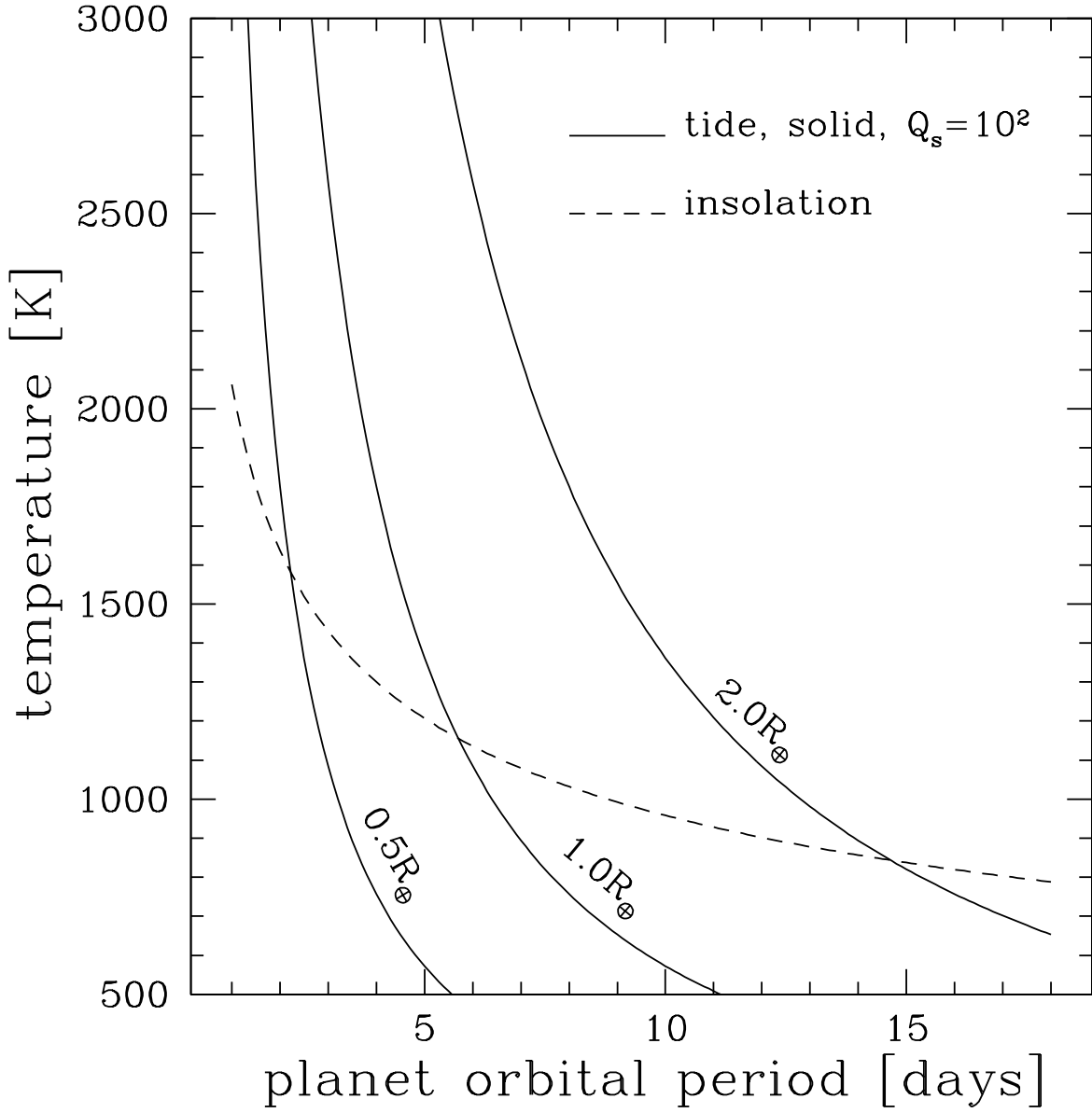


Fig. 3.— Effective temperature due to tidal heating (solid lines) or insolation (dashed line). Solid lines show temperature determined from eq.21, while dashed lines represent the equilibrium temperature due to insolation, and reradiation over  $4\pi$  steradians. Tidal heating determines the temperature when the solid line is above the dashed line.

Tidal heating dominates the temperature structure, even at the surface, when the temperature in eq.21 is larger than the equilibrium temperature implied by insolation. Figure 3 compares these temperatures for  $Q_s = 10^2$ , for a zero-albedo surface with uniform temperature over the entire surface. Clearly for  $Q_s = 10^2$ , Earth-sized objects are raised above the melting temperature at zero pressure ( $T_{\text{melt}} \lesssim 2000$  K) for either iron or rock compositions inside a critical orbital period, which we estimate as

$$P_{\text{p,melt}} = 3.7 \text{ days} \left( \frac{2000 \text{ K}}{T_{\text{melt}}} \right)^{4/5} \left( \frac{\rho_s}{5 \text{ g cm}^{-3}} \right)^{2/5} \times \left( \frac{R_s}{R_{\oplus}} \right) \left( \frac{10^2}{Q_s} \right)^{1/5}. \quad (22)$$

Hence, at the orbital periods of the hot Jupiters (a few days), satellites Earth size or larger are completely molten for  $Q_s = 10^2$ .

Vigorous tidal heating has consequences for the satellite magnetic field. Fluid motions in the conducting core driven by tidal heating may generate magnetic fields through dynamo action. Stevenson et al. (1983) derive a magnetic field for the Earth by equating the energy available for dynamo generation to the Ohmic dissipation rate of the field, which yields the scaling  $B^2 \propto \dot{E}$ , where  $\dot{E}$  is the tidal heating rate. As the dissipation rate in eq.20 is larger than the Earth’s cooling rate by a factor  $\sim 10^9/Q_s$ , our results imply that fields larger than Earth’s can be created if  $Q_s < 10^9$ . Intrinsic magnetic field can decrease the mass loss rate of the atmosphere. Magnetic interaction between the satellite and planetary magnetosphere may be of the Jupiter-Ganymede type (Kivelson et al. 1997) in which the standoff distance is determined mainly by magnetic stresses.

Studies of habitability of satellites around gas giants (e.g., Reynolds et al. 1987; Williams et al. 1997; Scharf 2006) have invoked tidal heating to maintain plate tectonics, create sub-surface oceans, etc. They computed satellite tidal heating for finite eccentricity, assuming it to be a free parameter, or pumped to large values by interaction with another satellite. The mechanism for tidal heating in this paper, periodic eccentricity forced by the star, is likely unimportant for satellite habitability, since it is only large for systems very close to the star, where water is already in liquid or vapor form. Tidal heating is of increasing importance, relative to insolation, for host stars of smaller mass since the stellar luminosity drops rapidly.

## 5. Energetics

Tidal dissipation in the satellite takes energy out of the satellite orbit and deposits it in the satellite body as heat. In this section we show that the energy lost from the satellite

orbit by heating is exactly balanced by energy gain from work by the stellar gravity. The tidal friction acts to create a lag in the satellite orbital velocity. This lag has the correct phase to allow energy input to the satellite’s orbit, and at a rate sufficient to power the tidal dissipation in the satellite. Ultimately, the energy reservoir for tidal heating of the satellite is then the star-planet orbit.

Tidal friction acts to exert a velocity dependent force on the reduced mass of the satellite-planet system given by (Mardling & Lin 2002, eq.4)

$$a_r^{(\text{TF})} = -\gamma_0 \dot{r}_s \quad (23)$$

$$a_\phi^{(\text{TF})} = -\gamma_0 r_s (\dot{\phi}_s - \Omega_s) \quad (24)$$

where  $\gamma_0 = (9/5)(n_s h_s / Q_s)(m_p / m_s)(R_s / a_s)^5$  is the frictional drag coefficient. Again we choose a circular background orbit plus small perturbations as in eq.5, 6 and 7. Eq.23 and 24 become

$$a_r^{(\text{TF})} = -\gamma_0 \delta \dot{r}_s \quad (25)$$

$$a_\phi^{(\text{TF})} = -\gamma_0 a_s (n_s - \Omega_s + \delta \dot{\phi}_s). \quad (26)$$

Assuming that the satellite spin has already reached the synchronous state, we may set  $\Omega_s = n_s$  in eq.26.

Adding the tidal friction acceleration in eq.25 and 26 into eq.8 and 9, and again ignoring the constant radial force term, we find equations which have the form of a damped, driven oscillator. In the limit  $\gamma_0 \ll n_s$  we find the solutions

$$\frac{\delta \dot{r}_s}{a_s} = 2 \left( \frac{n_p^2}{n_s} \right) \sin [2(n_s - n_p)t + 2\phi_{s0}] + \frac{35}{6} \gamma_0 \left( \frac{n_p}{n_s} \right)^2 \cos [2(n_s - n_p)t + 2\phi_{s0}] \quad (27)$$

$$\delta \dot{\phi}_s = \frac{11}{4} \left( \frac{n_p^2}{n_s} \right) \cos [2(n_s - n_p)t + 2\phi_{s0}] - \frac{173}{24} \gamma_0 \left( \frac{n_p}{n_s} \right)^2 \sin [2(n_s - n_p)t + 2\phi_{s0}]. \quad (28)$$

The first terms in eq.27 and 28 agree with eq.10 and 11. The second terms proportional to  $\gamma_0$  describe a lag in the satellite orbit due to tidal friction.

The energy lost from the orbit is found by computing the work done by the tidal friction force. Since the tidal friction force already contains the small parameter  $\gamma_0 \ll n_s$ , we can ignore terms of order  $\gamma_0$  in eq.27 and 28. For reduced mass  $\mu_s = M_s M_p / (M_s + M_p) \simeq M_s$ , and time averaging so that  $\cos^2, \sin^2 \rightarrow 1/2$ , we find

$$\frac{\dot{E}_{\text{s,orb}}^{(\text{TF})}}{\mu_s} = -\gamma_0 \left( 3\delta \dot{r}_s^2 + a_s^2 \delta \dot{\phi}_s^2 \right) = - \left( \frac{313}{32} \right) \gamma_0 a_s^2 \left( \frac{n_p}{n_s} \right)^2. \quad (29)$$

This is just a rederivation of eq.18.

The increase in satellite orbital energy due to work done by the stellar gravitational force in eq.8 and 9 is

$$\frac{\dot{E}_{s,\text{orb}}^{(\text{star})}}{\mu_s} = \frac{3}{2} n_p^2 a_s \left( \delta \dot{r}_s \cos [2(n_s - n_p)t + 2\phi_{s0}] - a_s \delta \dot{\phi}_s \sin [2(n_s - n_p)t + 2\phi_{s0}] \right). \quad (30)$$

The terms independent of  $\gamma_0$  in eq.27 and 28 time average to zero when inserted in eq.30. Inserting the terms proportional to  $\gamma_0$  into and time averaging we find

$$\frac{\dot{E}_{s,\text{orb}}^{(\text{star})}}{\mu_s} = \left( \frac{313}{32} \right) \gamma_0 a_s^2 \left( \frac{n_p}{n_s} \right)^2, \quad (31)$$

exactly equal and opposite to the energy lost to tidal heating in eq.29.

We have found that energy lost from the orbit is replenished by work done on the orbit by the stellar tidal force. Hence the satellite orbit is stable on long timescales due to tidal effects arising from dissipation in the satellite. The energy reservoir powering the tidal dissipation in the satellite must then come from the star-planet orbit. Given the large dissipation rate in eq.20, the star-planet orbital energy may undergo significant evolution for satellite  $Q_s \sim 10^2$ . However, for the more likely case of  $Q_s \gg 10^2$  for a molten satellite, the star-planet orbital evolution would be negligible.

## 6. Evaporative mass loss

The large tidal heating rates in eq.19 and 20, large insolation from the parent star, and low escape speed raise the possibility of enhanced rates of mass loss from the satellites studied in this paper. The rates of mass loss are difficult to predict, and may depend on many factors such as stellar wind or magnetospheric erosion of the atmosphere, and the flux of vapor from the surface to the exosphere. Here we discuss one possible scenario, of thermal evaporation from the solid surface of a satellite with a negligible atmosphere (such as the Moon or Mercury).

Cameron (1985) and Fegley & Cameron (1987) discussed evaporative mass loss from Mercury in the protoplanetary nebula as a means of understanding Mercury's high mean density. The high temperatures in the nebula allow enhanced rates of evaporation of low(er) density silicates from the solid surface into gaseous form. If this vapor is removed from the atmosphere, the mass fraction of iron in the body, and hence the mean density, will increase. Due to the exponential temperature dependence, this process is only effective

at high temperatures. We find that for surface temperatures due to tidal heating, or the equilibrium temperature due to insolation, that significant mass loss can occur. We will follow the simple estimate in Cameron (1985) to estimate the importance of this process.

The number flux for thermal evaporation from a surface, in  $\text{cm}^{-2} \text{s}^{-1}$ , is given by

$$\phi = \frac{P_{\text{eq}}(T)}{\sqrt{2\pi m k_b T}}, \quad (32)$$

where  $P_{\text{eq}}(T) = P_0 10^{-T_0/T}$  is the equilibrium vapor pressure above the surface,  $m$  is the mean molecular weight (in grams) of the products,  $T$  is the temperature, and  $P_0$  and  $T_0$  describe the equilibrium pressure. Cameron (1985) considers an illustrative example of  $\text{MgSiO}_3(\text{liq}) = \text{Mg}(\text{gas}) + \text{SiO}(\text{gas}) + \text{O}_2(\text{gas})$  (see Fegley & Cameron (1987) for more detailed calculation of silicate magmas of chondritic composition), for which  $P_0 = 10^{13.176} \text{ dyne cm}^{-2}$ ,  $T_0 = 24605 \text{ K}$ , and  $m = 5.5 \times 10^{-23} \text{ g}$ . If the surface temperature is near the equilibrium temperature for a zero albedo surface (neglecting tidal heating), radiating with the same efficiency over  $4\pi$ <sup>1</sup>, the equilibrium temperature for orbital radius for a solar type star is  $T = 5777 \text{ K} (R_\odot/2a_p)^{1/2} = 1300 \text{ K} (10R_\odot/a_p)^{1/2}$ . As seen in figure 3, this temperature including only insolation, and ignoring tidal heating, may underestimate the satellite surface temperature close to the star. Approximating the exponential as a power law (valid for small changes in  $a_p$ ), we find a mass flux

$$m\phi \simeq 1.3 \times 10^{-11} \text{ g cm}^{-2} \text{ s}^{-1} \left( \frac{10R_\odot}{a_p} \right)^{21.5} \quad (33)$$

from the surface. If the vapor is lost from the atmosphere, this translates into a mass loss rate

$$\dot{M}_s = 3.3 \times 10^{-4} M_\oplus \text{ Gyr}^{-1} \left( \frac{10R_\odot}{a_p} \right)^{21.5} \left( \frac{R_s}{R_\oplus} \right)^2. \quad (34)$$

In 5 Gyr, an Earth mass can be lost for  $a_p \lesssim 7.4R_\odot$ , where  $T \gtrsim 1500 \text{ K}$ . Hence satellites originally of Earth-size, and hence detectable by their transit signature with Kepler, may become undetectable due to erosion. Eq.34 implies the rate of decrease of the radius is independent of radius. Erosion does not slow for small satellites.

---

<sup>1</sup>We note that if the atmosphere is thin (surface pressure  $\ll 1 \text{ bar}$ ), the day-side temperature may be larger by a factor of  $2^{1/2}$ . This will cause the critical  $a_p$  for large mass loss to increase by a factor of 2, and the critical planet orbital period to increase by a factor  $2^{3/2}$ . Hence a critical orbital period inside of which large mass loss occurs depends sensitively on the  $2^{1/2}$ , since it occurs in an exponent. Our assumption of equal day-night temperatures is conservative.

Our estimates give an upper limit to the erosion rate, and show that large mass loss can occur over the lifetime of the satellite. The exponential dependence on surface temperature suggests a critical orbital radius (of planet around the star) inside of which satellites can be significantly eroded. Our conservative estimate finds the critical orbital period to be around 2.4 days (6.6 days using the day-side equilibrium temperature).

## 7. Conclusions

In this paper we have studied the orbits, tidal heating and evaporative mass loss rates from Earth-sized satellites orbiting hot Jupiters. We have found four main results. First, periodic variations in the satellite’s orbit due to the star induce large rates of tidal heating (see eq.19 and 20), likely melting all but perhaps a thin surface layer. Second, we have clarified that orbital decay due to tides raised in the planet by the satellite may not destroy these massive satellites as gas giants are so weakly dissipative at the forcing frequencies of interest. Next, we found that the satellite orbit does not evolve secularly due to tidal dissipation in the satellite, as the stellar gravity does work at a rate to keep the orbital energy constant. Lastly, the estimated upper limit to mass loss indicates a critical orbital period around a Sun-like star (2.4 days for a thick atmosphere, 6.6 days for a thin atmosphere) for Earth-like satellites orbiting a hot Jupiter. Inside this critical period significant erosion can occur, reducing satellites to a size undetectable by upcoming transit observations with the Kepler satellite. Outside the critical period, evaporative mass loss will become negligible so that detectable Earth size planets can survive.

We thank Ken Seidelmann for interesting discussions on lunar theory. TAC was supported by a NASA Graduate Student Researchers Program Fellowship through the Langley Research Center and a Virginia Space Grant Consortium Graduate Research Fellowship. RM received support through a Virginia Space Grant Consortium Undergraduate Scholarship. PA is an Alfred P. Sloan Fellow, and also acknowledges support from the University of Virginia Fund for Excellence in Science and Technology. REJ acknowledges support from NASA’s Planetary Atmospheres Program. We thank the anonymous referee for constructive comments which improved this paper significantly.

## REFERENCES

Arras, P., & Bildsten, L. 2006, *ApJ*, 650, 394

- Ballester, G. E., Sing, D. K., & Herbert, F. 2007, *Nature*, 445, 511
- Barnes, J. W., & O'Brien, D. P. 2002, *ApJ*, 575, 1087
- Barnes, J. W., & Fortney, J. J. 2004, *ApJ*, 616, 1193
- Basri, G., Borucki, W. J., & Koch, D. 2005, *New Astronomy Review*, 49, 478
- Borucki, W., et al. 2004, *Stellar Structure and Habitable Planet Finding*, 538, 177
- Brouwer, D., & Clemence, G. M. 1961, New York: Academic Press, 1961,
- Brown, T. M., Charbonneau, D., Gilliland, R. L., Noyes, R. W., & Burrows, A. 2001, *ApJ*, 552, 699
- Burns, J. A. 1973, *Nature*, 242, 23
- Burns, J. A. 1977, *IAU Colloq. 28: Planetary Satellites*,
- Cameron, A. G. W. 1985, *Icarus*, 64, 285
- Canup, R. M., & Ward, W. R. 2006, *Nature*, 441, 834
- Charbonneau, D., Brown, T. M., Noyes, R. W., & Gilliland, R. L. 2002, *ApJ*, 568, 377
- Domingos, R. C., Winter, O. C., & Yokoyama, T. 2006, *MNRAS*, 373, 1227
- Doyle, L. R., & Deeg, H.-J. 2004, *Bioastronomy 2002: Life Among the Stars*, 213, 80
- Dziewonski, A. M., & Anderson, D. L. 1981, *Physics of the Earth and Planetary Interiors*, 25, 297
- Efroimsky, M., & Lazarian, A. 2000, *MNRAS*, 311, 269
- Fegley, B., & Cameron, A. G. W. 1987, *Earth and Planetary Science Letters*, 82, 207
- Goldreich, P., & Soter, S. 1966, *Icarus*, 5, 375
- Goldreich, P., & Nicholson, P. D. 1977, *Icarus*, 30, 301
- Han, C., & Han, W. 2002, *ApJ*, 580, 490
- Jeffreys, H. 1970, Cambridge: University Press, 1970,
- Johnson, R. E., & Huggins, P. J. 2006, *PASP*, 118, 1136

- Kivelson, M. G., Khurana, K. K., Coroniti, F. V., Joy, S., Russell, C. T., Walker, R. J., Warnecke, J., Bennett, L., & Polanskey, C. 1997, *Geophys. Res. Lett.*, 24, 2155
- Mardling, R. A., & Lin, D. N. C. 2002, *ApJ*, 573, 829
- Masursky, H., Schaber, G. G., Soderblom, L. A., & Strom, R. G. 1979, *Nature*, 280, 725
- Ogilvie, G. I., & Lin, D. N. C. 2004, *ApJ*, 610, 477
- Peale, S. J., & Cassen, P. 1978, *Icarus*, 36, 245
- Peale, S. J., Cassen, P., & Reynolds, R. T. 1979, *Science*, 203, 892
- Peale, S. J. 1999, *ARA&A*, 37, 533
- Pont, F., et al. 2007, *A&A*, 476, 1347
- Redfield, S., Endl, M., Cochran, W. D., & Koesterke, L. 2008, *ApJ*, 673, L87
- Reynolds, R. T., McKay, C. P., & Kasting, J. F. 1987, *Advances in Space Research*, 7, 125
- Roult, G., Rosat, S., Clévéde, E., Millot-Langet, R., & Hinderer, J. 2006, *Journal of Geodynamics*, 41, 345
- Sartoretti, P., & Schneider, J. 1999, *A&AS*, 134, 553
- Scharf, C. A. 2006, *ApJ*, 648, 1196
- Simon, A., Szatmáry, K., & Szabó, G. M. 2007, *A&A*, 470, 727
- Stevenson, D. J., Spohn, T., & Schubert, G. 1983, *Icarus*, 54, 466
- Szabó, G. M., Szatmáry, K., Divéki, Z., & Simon, A. 2006, *A&A*, 450, 395
- Vidal-Madjar, A., Lecavelier des Etangs, A., Désert, J.-M., Ballester, G. E., Ferlet, R., Hébrard, G., & Mayor, M. 2003, *Nature*, 422, 143
- Vidal-Madjar, A., et al. 2004, *ApJ*, 604, L69
- Ward, W. R., & Reid, M. J. 1973, *MNRAS*, 164, 21
- Williams, D. M., Kasting, J. F., & Wade, R. A. 1997, *Nature*, 385, 234
- Wisdom, J. 2004, *AJ*, 128, 484
- Wu, Y. 2003, *Scientific Frontiers in Research on Extrasolar Planets*, 294, 213

Wu, Y. 2005, ApJ, 635, 688